CENTRAL AND INSCRIBED ANGLES

A central angle is an angle whose vertex is the center of a circle and whose sides intersect the circle. The degree measure of a central angle is equal to the degree measure of its intercepted arc. For the circle at right with center C, ∠ACB is a central angle.

An **INSCRIBED ANGLE** is an angle with its vertex on the circle and whose sides intersect the circle. The arc formed by the intersection of the two sides of the angle and the circle is called an **INTERCEPTED ARC**. ∠ADB is an inscribed angle, \( \widehat{AB} \) is an intercepted arc.

The **INSCRIBED ANGLE THEOREM** says that the measure of any inscribed angle is half the measure of its intercepted arc. Likewise, any intercepted arc is twice the measure of any inscribed angle whose sides pass through the endpoints of the arc.

\[ m\angle ADB = \frac{1}{2} \widehat{AB} \text{ and } \widehat{AB} = 2m\angle ADB \]

**Example 1**
In \( \odot P \), \( m\angle CPQ = 70^\circ \). Find \( m\widehat{CQ} \) and \( m\widehat{CRQ} \).

\[ m\widehat{CQ} = m\angle CPQ = 70^\circ \]
\[ m\widehat{CRQ} = 360^\circ - m\widehat{CQ} = 360^\circ - 70^\circ = 290^\circ \]

**Example 2**
In the circle shown below, the vertex of the angles is at the center. Find \( x \).

Since \( \widehat{AB} \) is the diameter of the circle, \( x + 2x = 180^\circ \Rightarrow 3x = 180^\circ \Rightarrow x = 60^\circ \)

**Example 3**
Solve for \( x \).

Since \( m\widehat{BAC} = 220^\circ \), \( m\widehat{BC} = 360^\circ - 220^\circ = 140^\circ \). The \( m\angle BAC \) equals half the \( m\widehat{BC} \) or \( 70^\circ \).
Find each measure in \( \odot P \) if \( \angle WPX = 28^\circ \), \( \angle ZPY = 38^\circ \), and \( \overline{WZ} \) and \( \overline{XV} \) are diameters.

1. \( \angle YZ \)  
2. \( \angle WX \)  
3. \( \angle VPZ \)  
4. \( \angle VWX \)  
5. \( \angle XPY \)  
6. \( \angle XY \)  
7. \( \angle XWY \)  
8. \( \angle WZX \)

In each of the following figures, \( O \) is the center of the circle. Calculate the values of \( x \) and justify your answer.

9. \( \angle 136^\circ \)
10. \( \angle 146^\circ \)
11. \( \angle 62^\circ \)
12. \( \angle 49^\circ \)
13. \( \angle 100^\circ \)
14. \( \angle 150^\circ \)
15. \( \angle 55^\circ \)
16. \( \angle 35^\circ \)
17. \( \angle 27^\circ \)
18. \( \angle 91^\circ \)
19. \( \angle 35^\circ \)
20. \( \angle 101^\circ \)

Answers

1. \( 38^\circ \)  
2. \( 28^\circ \)  
3. \( 28^\circ \)  
4. \( 180^\circ \)  
5. \( 114^\circ \)  
6. \( 114^\circ \)  
7. \( 246^\circ \)  
8. \( 332^\circ \)  
9. \( 68^\circ \)  
10. \( 73^\circ \)  
11. \( 98^\circ \)  
12. \( 124^\circ \)  
13. \( 50^\circ \)  
14. \( 55^\circ \)  
15. \( 18^\circ \)  
16. \( 27^\circ \)  
17. \( 55^\circ \)  
18. \( 77^\circ \)  
19. \( 35^\circ \)  
20. \( 50^\circ \)