CONES AND SPHERES

For a CONE, the formula for volume is the same as that for a pyramid except now the base is a circle.

\[ V = \frac{1}{3} \pi r^2 h \]

The lateral surface area is the same as the area of the sector found by unrolling the cone.

\[ LA = \pi r l \]

In these formulas, \( r \) represents the radius of the base, \( h \) is the height and \( l \) is the slant height.

For a SPHERE with radius \( r \), the volume and surface area is found using:

\[ V = \frac{4}{3} \pi r^3 \quad \text{and} \quad SA = 4\pi r^2 \]

Example 1

Find the volume and lateral surface area of the cone at right. Note that \( r = 3 \) cm.

\[ V = \frac{1}{3} \pi 3^2 \cdot 4 = 12\pi \approx 37.70 \text{ cm}^3 \]

\[ LA = \pi \cdot 3 \cdot 5 = 15\pi \approx 47.12 \text{ cm}^2 \]

Example 2

A cone with volume 63.62 cubic units has a height of 3 units. What is the radius of the cone and the lateral surface area?

First use the formula for volume to find the radius.

\[ \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^2 \cdot 3 = 63.62 \quad \text{Substituting} \]

\[ \pi r^2 = 63.62 \quad \text{Simplifying} \]

\[ r^2 = \frac{63.62}{\pi} \]

\[ r = \sqrt{\frac{63.62}{\pi}} = 4.5 \text{ units} \quad \text{Solve for } r \]

>>Problem continues next page.>>
To find the lateral area we need the slant height. Notice that the radius, height, and slant height form a right triangle so we use \( r^2 + h^2 = l^2 \) (the Pythagorean theorem.)

\[
\begin{align*}
4.5^2 + 3^2 &= l^2 & \text{Substituting} \\
29.25 &= l^2 & \text{Simplifying} \\
l &\approx 5.41 \text{ units} & \text{Solving} \\
\text{LA} &= \pi rl = \pi \cdot 4.5 \cdot 5.41 & \text{Substituting}
\end{align*}
\]

\[\text{LA} \approx 76.48 \text{ square units}\]

**Example 3**

Find the volume and surface area of the sphere at right.

\[
\begin{align*}
V &= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \cdot 2^3 = \frac{32\pi}{3} \approx 33.51 \text{ ft}^3 \\
\text{SA} &= 4\pi \cdot 2^2 = 16\pi \approx 50.27 \text{ ft}^2
\end{align*}
\]

**Example 4**

A sphere has a volume of \( 972\pi \). Find the surface area.

First use the formula for volume and solve for the radius. Then substitute the value for the radius into the formula for surface area.

\[
\begin{align*}
V &= \frac{4}{3} \pi r^3 = 972\pi & \text{Substituting} \\
4\pi r^3 &= 2916\pi & \text{Solving} \\
r^3 &= \frac{2916\pi}{4\pi} = 729 \\
r &= \sqrt[3]{729} = 9 & \text{Solving} \\
\text{SA} &= 4\pi r^2 = 4\pi \cdot 9^2 & \text{Substituting} \\
\text{SA} &= 324\pi & \text{Simplifying}
\end{align*}
\]
Use the given information to find the volume of the cone.

1. radius = 1.5 in, height = 4 in
2. diameter = 6 cm, height = 5 cm
3. base area = \(25\pi\), height = 3
4. base circ. = \(12\pi\), height = 10
5. diameter = 12, slant height = 10
6. lateral area = \(12\pi\), radius = 1.5

Use the given information to find the lateral area of the cone.

7. radius = 8 in, slant height = 1.75 in
8. slant height = 10 cm, height = 8 cm
9. base area = \(25\pi\), slant height = 6
10. radius = 8 cm, height = 15 cm
11. volume = \(100\pi\), height = 5
12. volume = \(36\pi\), radius = 3

Use the given information to find the volume of the sphere.

13. radius = 10 cm
14. diameter = 10 cm
15. circumference of great circle = \(12\pi\)
16. surface area = \(256\pi\)
17. circumference of great circle = 20 cm
18. surface area = 100

Use the given information to find the surface area of the sphere.

19. radius = 5 in
20. diameter = 12 in
21. circumference of great circle = \(14\pi\)
22. volume = 250
23. circumference of great circle = \(\pi\)
24. volume = \(\frac{9\pi}{2}\)
Answers
1. $3\pi \approx 9.42$ in$^3$
2. $15\pi \approx 47.12$ cm$^3$
3. $25\pi \approx 78.54$ u$^3$
4. $120\pi \approx 376.99$ u$^3$
5. $96\pi \approx 301.59$ u$^3$
6. $\approx 18.51$ u$^3$
7. $14\pi \approx 43.98$ in$^2$
8. $60\pi \approx 188.50$ cm$^2$
9. $30\pi \approx 94.25$ u$^2$
10. $136\pi \approx 427.26$ cm$^2$
11. $\approx 224.35$ u$^2$
12. $116.58$ u$^2$
13. $\frac{4000\pi}{3} \approx 4188.79$ cm$^3$
14. $\frac{500\pi}{3} \approx 523.60$ cm$^3$
15. $288\pi \approx 904.79$ u$^3$
16. $\frac{2048\pi}{3} \approx 2144.66$ u$^3$
17. $\approx 135.09$ u$^3$
18. $\approx 94.03$ u$^3$
19. $100\pi \approx 314.16$ u$^2$
20. $144\pi \approx 452.39$ u$^2$
21. $\approx 62.39$ u$^2$
22. $\approx 191.91$ u$^2$
23. $\pi \approx 3.14$ u$^2$
24. $9\pi \approx 28.27$ u$^2$