6. Factoring

The first step in factoring any expression is to look for a greatest common factor. For example, in the expression $2x^3y + 8x^2y^2$, the greatest common factor is $2x^2y$, therefore $2x^3y + 8x^2y^2 = 2x^2y(x + 4y)$.

Area models are useful tools for factoring.

Example: Factor $2x^2 + x – 6$.

Solution: Sketch an area model with four sections.

Write the quadratic ($2x^2$) and constant ($–6$) terms along one diagonal.

The products of the diagonals must be equivalent. Therefore, identify two terms whose product equals the product of the constant terms and whose sum is the linear term. In this case, the two terms are $4x$ and $–3x$. Write these terms as the other diagonal.

Write the dimensions of the rectangle by determining the greatest common factor in each row and column.

Write the complete factorization. $2x^2 + x – 6 = (2x – 3)(x + 2)$

**Special Factoring Patterns**

- **Difference of Two Squares:** $a^2 – b^2 = (a + b)(a – b)$
- **Sum of Two Cubes:** $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$
- **Difference of Two Cubes:** $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$

Solution to problem 2-159:

a. $x^4 – 81y^4$

$$
(x^2)^2 – (9y^2)^2
$$

$$
(x^2 + 9y^2)(x^2 – 9y^2)
$$

$$
(x^2 + 9y^2)(x + 3y)(x – 3y)
$$

c. $10x^2 – 33x – 7$

$$
\begin{array}{c|c|c|c}
-35x & -7 & -35x & -7 \\
10x^2 & 2x & 2x & 10x^2 & 2x \\
5x^2 & 2 & 5x^2 & 2x & 5x^2 & 2x \\
\end{array}
$$

$$(2x – 7)(5x + 1)$$

d. $5x^2 – 34x + 24$

$$
\begin{array}{c|c|c|c}
-30x & 24 & -30x & 24 \\
5x^2 & -4x & 5x^2 & -4x \\
5x – 4 & x & 5x – 4 & x \\
\end{array}
$$

$$(5x – 4)(x – 6)$$
Completely factor each of the following expressions.

1. \(42a^3b^3 + 24a^2b^5\)  
2. \(24\pi^4h - 20\pi^2h^4\)  
3. \(x^2 - 12x + 36\)  
4. \(x^3 + 1\)  
5. \(x^2 + 5x + 6\)  
6. \(b^2 - 16\)  
7. \(x^2 - x - 42\)  
8. \(9a^2 + 24a + 16\)  
9. \(2x^2 - 17x + 35\)  
10. \(6m^4n^2 - 7mn^4 + 7m^2n^2\)  
11. \(49y^2 - 21y + 2\)  
12. \(8x^3 - 125\)  
13. \(10x^2 - 29x + 21\)  
14. \(6z^2 - 12z + 18\)  
15. \(4w^2 - 169\)  
16. \(30t^{11}u^4 + 15t^{10}u^7v^3 - 35t^9u^8v^5\)  
17. \(16h^2 + 40hk + 25k^2\)  
18. \(8p^3 + 27q^3\)  
19. \(7c^4 - 7\)  
20. \(8r^3s + 24r^2s^2 - 80rs\)  
21. \(42x^2 - 175x + 28\)  
22. \(6d^4e^3 + 8d^2e^5 - 2d^3e^2 + 6d^2e^2\)  
23. \(-10x^2 + 31x - 15\)  
24. \(g^6 - j^6\)

**Answers**

1. \(6a^2b^3(7a + 4b^2)\)  
2. \(4\pi^2h(6r^2 - 5h^3)\)  
3. \((x - 6)^2\)  
4. \((x + 1)(x^2 - x - 1)\)  
5. \((x + 3)(x + 2)\)  
6. \((b + 4)(b - 4)\)  
7. \((x + 6)(x - 7)\)  
8. \((3a + 4)^2\)  
9. \((2x - 7)(x - 5)\)  
10. \(mn^2(6m^3 - 7n^2 + 7m)\)  
11. \((7x - 1)(7x - 2)\)  
12. \((2x - 5)(x^2 + 10x + 25)\)  
13. \((2x - 3)(5x - 7)\)  
14. \(2(z - 3)^2\)  
15. \((2w + 13)(2w - 13)\)  
16. \(5t^9u^6v^3(6t^2v^3 + 3tu - 7u^2v^2)\)  
17. \((4h + 5k)^2\)  
18. \((2p + 3q)(4p^2 - 6p + 9q^2)\)  
19. \(7(c + 1)(c - 1)(c^2 + 1)\)  
20. \(8rs(r - 2)(r + 5)\)  
21. \(7(6x - 1)(x - 4)\)  
22. \(2d^2e^2(3d^2e^2 + 4e^3 - d + 3)\)  
23. \((2x - 5)(3 - 5x)\) or \((5x - 3)(2 - 2x)\)  
24. \((g^3 + j^3)(g^3 - j^3) = (g + j)(g^2 - gj + j^2)(g - j)(g^2 + gj + j^2)\)