13. Inequalities

**Example 1:** Solve: \[|3x - 5| \leq 13.\]

*Solution:* Start by writing the inequality. \(-13 \leq 3x - 5 \leq 13\)

Add 5 to all three members. \(-8 \leq 3x \leq 18\)

Divide all three members by 3. \(-\frac{8}{3} \leq x \leq 6\)

The answer written in interval notation is \(\left[-\frac{8}{3}, 6\right]\).

Remember that if you multiply or divide an inequality by a negative number you must reverse the inequality. For example, solve \(-2x \geq 18, x \leq -9\).

Dividing both sides by \(-2\) reverses the inequality.

**Example 2:** Solve: \(x^2 - x > 6\)

*Solution:* As with equations, we need to have a polynomial equal to zero so that we can factor.

Subtract 6 from both sides. \(x^2 - x - 6 > 0\)

Factor: \((x - 3)(x + 2) > 0\)

Find the zero’s. \(x = 3, x = -2\)

Test the intervals to find where the quadratic is greater than zero.

Here the interval is positive for values less than \(-2\) or greater than \(3\). It is negative between \(-2\) and \(3\).

Express the solution using inequalities \(x < -2\) or \(x > 3\).
Solve the inequality. Write your answer in either interval or inequality notation.

1. \(|x| \geq -2\)
2. \(|x - 3| < 7\)
3. \(|x - 3| + 2 < 6\)
4. \(|2x + 1| + 4 \geq 7\)
5. \(x^2 + x - 6 > 0\)
6. \(x^2 - x - 12 \leq 0\)
7. \(6x^2 < 5x - 1\)
8. \((x - 2)(x - 3)(x - 4) > 0\)

Solve the inequality and graph the solution set.

9. \(|x - 1| < -3\)
10. \(|3x + 9| \geq 6\)

Example: Graph the solution \(y + 5 > 2x\).

Solution: Begin by treating the expression as an equality and sketch the graph. So begin by graphing \(y = 2x - 5\). Since the inequality is \(y \) greater than \(2x - 5\) the line is dotted because it is not included. Next you should choose a test point on one side of the line and plug it into the inequality. If it makes the inequality true, then shade that side of the graph. If it makes the inequality false, than shade the other side of the graph. A good point to use (as long as it is not on the graph) is \((0, 0)\). If we substitute \(x = 0\) and \(y = 0\) into the inequality, then it becomes \(0 + 5 > 10\), which is false so shade the other side of the line.

Graph the solution.

11. \(x - 2y < 4\)
12. \(x^2 + 5x \geq -6\)
13. \(2x + 3y \leq 6\)
14. \(x - y > -3\)
15. \(2x + y \leq 5\)
16. \(3x + 5y \leq 7\)