16. Logarithms

Example 1: Write as a sum or difference of multiples of logarithms: \( \log_b \sqrt[3]{x^2 y^3} \)

Solution: Use log properties to break down expression.

Quotient Rule: \( \log_b \frac{\sqrt{x}}{y^3} \)

Power Rule: \( \frac{1}{2} \log_b x - 3 \log_b y \)

Example 2: Solve: \( \ln(x + 3) - \ln(x - 2) = \ln 4 \)

Solution: Use quotient rule to combine terms. \( \ln \frac{x+3}{x-2} = \ln 4 \)

Take \( e^x \) on both sides. \( \frac{x+3}{x-2} = 4 \)

Solve the expression. \( x + 3 = 4x - 8 \)

\( 11 = 3x \)

\( x = \frac{11}{3} \)

Make sure the solution is in the domain of the expression. In this case the domain is \( x > 2 \). Since \( \frac{11}{3} > 2 \), it is a valid solution.

Write each expression as a sum or difference of multiples of logarithms.

1. \( \log_b x^2 y^3 \)
2. \( \log_b \frac{x^9}{y^7} \)
3. \( \log_b \frac{\sqrt[5]{x^6}}{x^2} \)
4. \( \ln \frac{x^3 y^2}{\sqrt[3]{c}} \)
5. \( \ln \frac{\sqrt[3]{x^4 + 3}}{x^5} \)
6. \( \ln \frac{1}{\sqrt[3]{6x^2 - 7x - 3}} \)

Write each expression as a single logarithm.

7. \( \log_5 \frac{5}{7} + \log_5 \frac{40}{22} \)
8. \( \log_2 \frac{32}{11} + \log_2 \frac{121}{16} - \log_2 \frac{4}{5} \)
9. \( \ln \left( 4x^2 - 9 \right) - \ln \left( 8x^3 - 27 \right) \)
10. \( 3 \ln \frac{a^2 b}{c^2} + 2 \ln \frac{b^2 c}{a^4} + 2 \ln \frac{abc}{2} \)

Solve each equation without using a calculator.

11. \( \log_3 (x + 1) = 2 \)
12. \( \log_5 (5x - 1) = -2 \)
13. \( \log_2 2x = \log_3 (x + 1) \)
14. \( \ln x + \ln (x - 2) = \ln (x + 4) \)
15. \( 2 \ln (x + 1) - \ln (x + 4) = \ln (x - 1) \)
16. \( \log_4 x + \log_4 (6x + 10) = 1 \)