7. Simplifying and Combining Radicals

Example: Simplify $\sqrt{288}$.

When you are asked to write a radical in simpler form, the real task is to reduce the number under the radical sign as much as possible by taking perfect squares outside the radical.

Solution: Simplify $\sqrt{288}$.

Find the factorization of 288, choosing perfect squares as factors whenever possible.

$288 = 4 \cdot 4 \cdot 9 \cdot 2$

$\sqrt{288} = \sqrt{4 \cdot 4 \cdot 9 \cdot 2}$

$\sqrt{288} = 2 \cdot 2 \cdot 3 \cdot \sqrt{2}$

$\sqrt{288} = 12\sqrt{2}$

Rewrite the following as radicals in simpler form.

1. $\sqrt{1200}$
2. $\sqrt{845}$
3. $\sqrt{a^3b^5}$
4. $\sqrt{x^7b^9}$

Example: Simplify $3\sqrt{125} - 2\sqrt{80}$.

Solution: First simplify each radical separately and then combine like terms.

$3\sqrt{125} = 3\sqrt{25\cdot 5} = 15\sqrt{5}$

$2\sqrt{80} = 2\sqrt{16\cdot 5} = 8\sqrt{5}$

$3\sqrt{125} - 2\sqrt{80} = 15\sqrt{5} - 8\sqrt{5} = 7\sqrt{5}$

Simplify the following expressions.

5. $\sqrt{5} + \sqrt{18}$
6. $\sqrt{54} + \sqrt{294}$

7. $\sqrt{18} + \sqrt{108} + \sqrt{50} + \sqrt{48}$
8. $(\sqrt{99})^2$

9. $(\sqrt{12})(\sqrt{75})$
10. $(\sqrt{20})(\sqrt{80})$

11. $(\sqrt{50})(\sqrt{98})$
Example: Rationalize $\frac{4}{\sqrt{3}}$.

Solution: To rationalize the denominator you need to multiply both numerator and denominator by the radical in the denominator. This is equivalent to multiplying by 1 so it does not change the value of the expression.

$$\frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

Rationalize the following expressions.

12. $\frac{3}{\sqrt{2}}$  
13. $\frac{5}{\sqrt{7}}$  
14. $\frac{a^2b^4}{\sqrt{a^5b^9}}$

Example: Combine and simplify $\frac{2}{\sqrt{3}} + \frac{1}{2\sqrt{3}}$.

Solution: To combine rational expressions you first need to find a common denominator and write the expression as one fraction. Then, you must rationalize the denominator.

$$\frac{2}{\sqrt{3}} + \frac{1}{2\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{2}{2} + \frac{1}{\sqrt{3}} \cdot \frac{1}{1} = \frac{5\sqrt{3}}{6}$$

Rewrite the following as single, rationalized expressions.

15. $\frac{3}{\sqrt{2}} + \frac{6}{\sqrt{2}}$  
16. $\frac{6}{\sqrt{8}} + \frac{3}{\sqrt{8}}$