There are $\binom{52}{5} = 2,598,960$ possible distinct poker hands.

**Royal flush:** 10-J-Q-K-A, all the same suit.

There are 4 royal flushes, so $P(\text{royal flush}) = \frac{4}{2,598,960} \approx 0.000038\%$

**Straight flush:** 7-8-9-10-J, any five in a row, all the same suit (A can be used before 2 or after K).

There are 10 possible straights in each suit, so 40 possible straight flushes in all. We must subtract the 4 royal flushes, so there are 36 ways to get a straight flush.

$P(\text{straight flush}) = \frac{36}{2,598,960} \approx 0.0014\%$

**Four of a kind:** 2-2-2-2-X, four of a number or face card, and any other card.

There are 13 ways to draw four cards of the same number and there are 48 possibilities for the fifth card, so there are $13 \cdot 48 = 624$ possible four of a kind hands.

$P(4 \text{ of a kind}) = \frac{624}{2,598,960} \approx 0.024\%$

**Full house:** 7-7-7-A-A, three of one kind and two of another.

There are $4 \cdot \binom{3}{3} = 12 \cdot \binom{4}{3}$ ways to get three of a kind from four of each value and there are 13 values, so there are $12 \cdot \binom{4}{3}$ ways to get three of a kind. Then there are $4 \cdot \binom{2}{2}$ ways to get two of a kind from four of each value and there are 12 remaining values, so there are $12 \cdot \binom{2}{2}$ ways to get the two of a kind. In all, there are $12 \cdot \binom{4}{3} \cdot 12 \cdot \binom{2}{2} = 3744$ ways to get a full house.

$P(\text{full house}) = \frac{3744}{2,598,960} \approx 0.144\%$

**Flush:** any five cards of the same suit, not all consecutive.

There are $\binom{13}{5}$ ways to get 5 cards from 13 of the same suit and there are 4 suits. We must subtract the 36 straight flushes and 4 royal flushes, so there are $4 \cdot \binom{13}{5} - 40 = 5108$ ways to get a flush. Then $P(\text{flush}) = \frac{5108}{2,598,960} = 0.197\%$
Straight: 3-4-5-6-7, any five in a row, a mixture of 2 or more suits.

Out of the numbers A through K, where A can be before 2 or after K, there are 10 possible straights. For each of these 10 possible straights, there are \(4^5\) possible combinations of suits, for a total of \(10 \cdot 4^5 = 10,240\) possible straights. This number includes 36 straight flushes and 4 royal flushes, so the total number of poker hands classified as straights are \(10,200\). \(P(\text{straight}) = \frac{10,200}{2,598,960} \approx 0.392\%\)

Three of a kind: 8-8-8-J-A, three of a number or face card, the other two different.

There are \(\binom{4}{3}\) ways to get three of a kind from four of each value and there are 13 values, so there are \(13 \cdot \binom{4}{3}\) ways to get three of a kind. Then there are \(\binom{12}{2}\) ways to get two differently valued cards from the remaining 12 values and, for each card, there are four possible suits, so there are a total of \(13 \cdot \binom{4}{3} \cdot \binom{12}{2} \cdot 4 \cdot 4 = 54,912\) ways to get the hand three of a kind. \(P(\text{three of a kind}) = \frac{54,912}{2,598,960} = 2.113\%\)

Two pair: 9-9-5-5-2, pairs of two different numbers or face cards, with one other card.

There are \(\binom{13}{2}\) ways to choose two values out of thirteen. Each of the two pairs chooses 2 suits from 4 or \(\binom{4}{2} \cdot \binom{4}{2}\). There are 11 values remaining for the 5th card, and each value has four suits. The total number of ways, then, to get the poker hand two pair is \(\binom{13}{2} \cdot (\binom{4}{2})^2 \cdot 11 \cdot 4 = 123,552\). \(P(\text{two pair}) = \frac{123,552}{2,598,960} \approx 4.754\%\)

Two of a kind: A-A-7-8-J, any pair with three random others that do not match.

There are \(\binom{13}{4}\) ways to get the pair. Then there are \(\binom{12}{3}\) ways to choose the values of the three remaining cards, and for each card there are four possible suits. So, the total number of ways to get the poker hand two of a kind is \(13 \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4 \cdot 4 = 1,098,240\). \(P(\text{two of a kind}) = \frac{1,098,240}{2,598,960} \approx 42.257\%\)

Bust: no matches, no runs of five in a row, different suits.

The probability of getting a bust is the sum of all the other probabilities subtracted from 1. \(P(\text{bust}) = 1 - \frac{1,420,012}{2,598,960} \approx 45.362\%\)